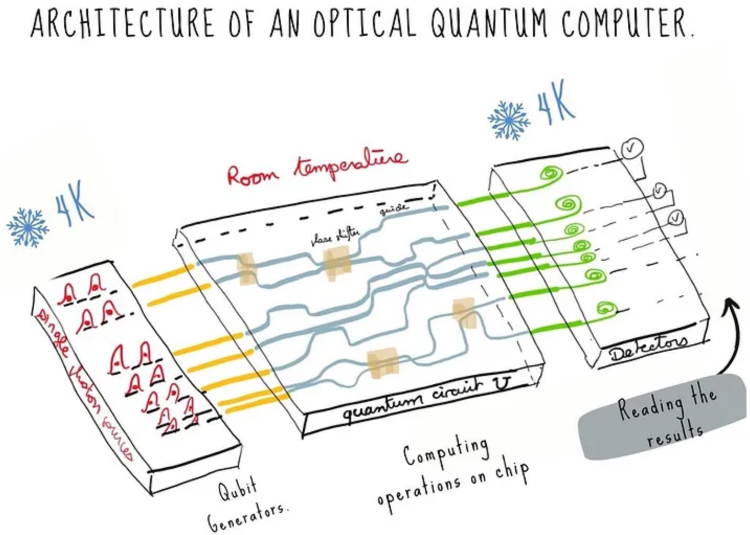
Finding Photonics Circuits with SMT Solvers

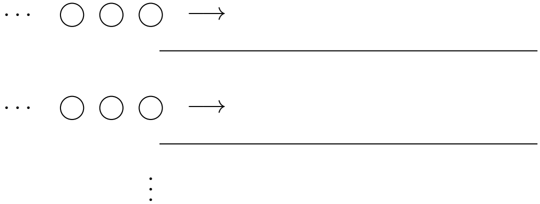
Marco Lewis Benoît Valiron

Université Paris-Saclay, CNRS, CentraleSupélec, ENS Paris-Saclay, Inria, Laboratoire Méthodes Formelles



From Quandela Medium Article: How to do computations on an optical quantum computer?

Wires, Phase Shifters, and Beam Splitters



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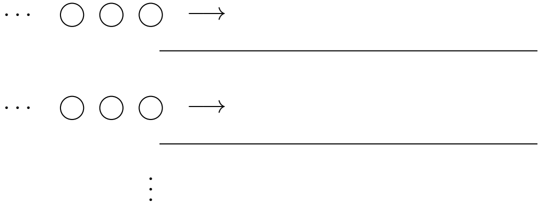
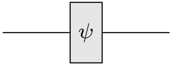


$e^{i\psi}$



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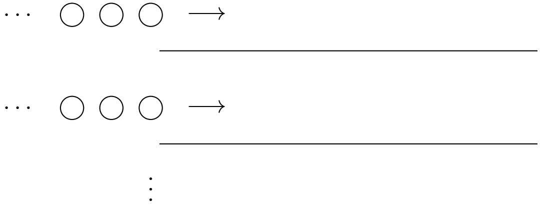

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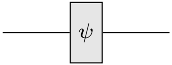
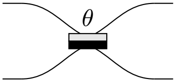
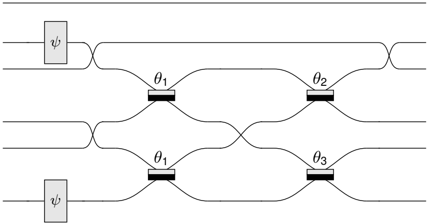


$$\begin{pmatrix} \cos(\theta) & ie^{-i\phi}\sin(\theta) \\ ie^{i\phi}\sin(\theta) & \cos(\theta) \end{pmatrix}$$



Photonics Circuits and Transfer Matrix



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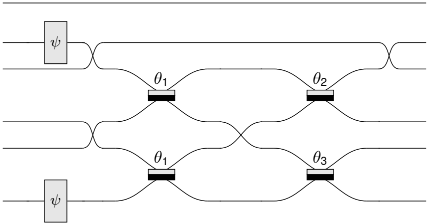

$$\iff$$

$$\hat{U} = \begin{pmatrix} u_{11} & u_{12} & \dots & u_{16} \\ u_{21} & u_{22} & \dots & u_{26} \\ \vdots & \vdots & \ddots & \vdots \\ u_{61} & u_{62} & \dots & u_{66} \end{pmatrix}$$

Fock States

Standard Fock States

$$|n_1, n_2, \ldots, n_m\rangle_{\mathcal{F}}$$

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$\hat{U}$ acts on single photons/creation operators ($\hat{a}_i^\dagger |\ldots n_i \ldots\rangle_{\mathcal{F}} \to \sqrt{n_i + 1} |\ldots n_i + 1 \ldots\rangle_{\mathcal{F}}$)

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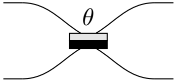
$(\hat{U})_{\mathcal{F}}$ acts on Fock basis $\mathcal{B}_{n,m}$

$$(\hat{U})_{\mathcal{F}} |n_1, \ldots, n_m\rangle_{\mathcal{F}} = \prod_{j=1}^{m} \frac{1}{\sqrt{n_j!}} \left(\sum_{i=1}^{m} \hat{U}_{ij} \hat{a}_i^\dagger \right)^{n_j} |0, \ldots, 0\rangle_{\mathcal{F}}$$

Simple Example



$\theta = \pi/4$, $\phi = \pi/2$

$$\hat{U} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

$|1, 0\rangle_{\mathcal{F}}$

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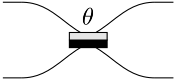

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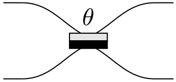

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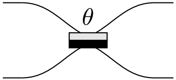

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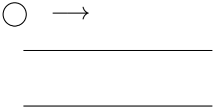
$$\begin{aligned} |1,1\rangle_{\mathcal{F}} &\to \frac{1}{2} |1,1\rangle_{\mathcal{F}} + \frac{\sqrt{2}}{2} |2,0\rangle_{\mathcal{F}} - \frac{\sqrt{2}}{2} |0,2\rangle_{\mathcal{F}} - \frac{1}{2} |1,1\rangle_{\mathcal{F}} \\ &= \frac{1}{\sqrt{2}} |2,0\rangle_{\mathcal{F}} - \frac{1}{\sqrt{2}} |0,2\rangle_{\mathcal{F}} \end{aligned}$$

Dual-Rail Encoding

$|0\rangle$



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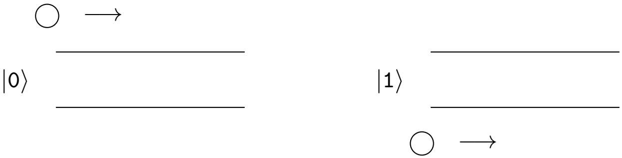

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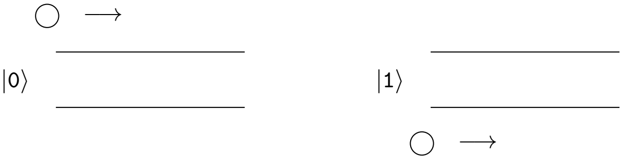

Computation Basis States ($\mathcal{C}_{qc} \subset \mathcal{B}_{n,m}$)

$$|0\rangle = |1,0\rangle_{\mathcal{F}}\,|\texttt{suffix}\rangle_{\mathcal{F}} \qquad |1\rangle = |0,1\rangle_{\mathcal{F}}\,|\texttt{suffix}\rangle_{\mathcal{F}}$$

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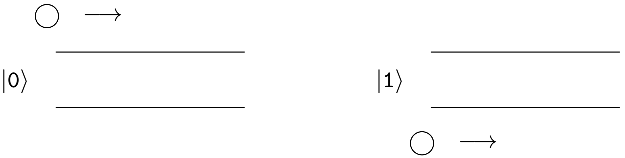

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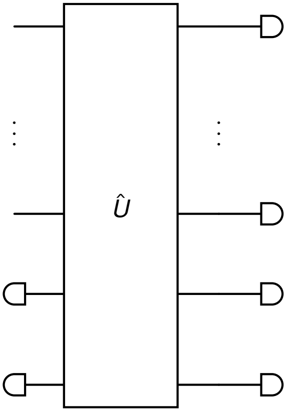
Example suffixes:

$$|\rangle_{\mathcal{F}}\,,\ |0,0\rangle_{\mathcal{F}}\,,\ |3\rangle_{\mathcal{F}}\,,\ |1,1\rangle_{\mathcal{F}}$$

Measurement Methods

Post-Selection



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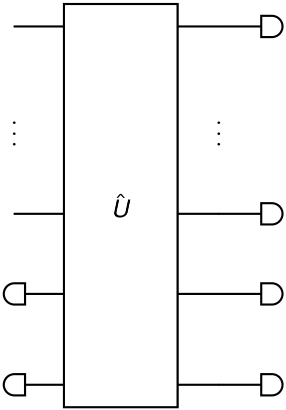
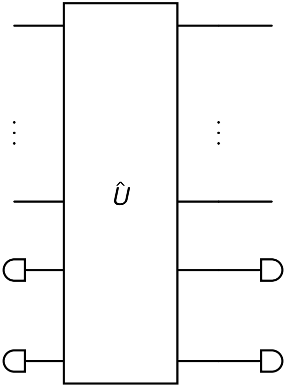


Heralded



Implementing Gates

Want to implement a quantum computing operation U on computation basis ($\mathcal{C}_{qc}$)

$$(\hat{U})_{\mathcal{F}} = \begin{array}{c} \\ |\mathcal{C}_{qc}|\left\{ \vphantom{\begin{matrix} a \\ b \end{matrix}} \right. \end{array} \begin{pmatrix} \overbrace{\alpha U}^{|\mathcal{C}_{qc}|} & M_1 \\ M_0 & M_2 \end{pmatrix},$$

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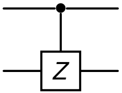
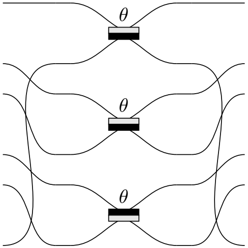
Success probability: $|\alpha|^2$ $(\alpha \in \mathbb{C})$

CZ Example - Post-select

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

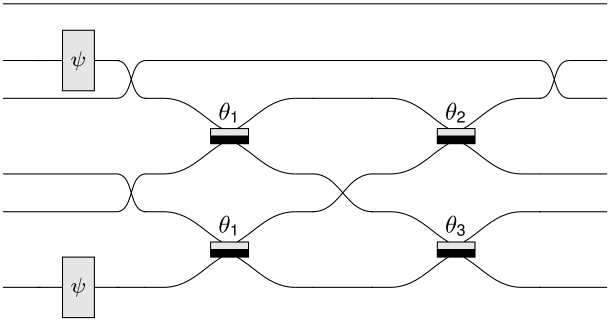


$$\frac{1}{3}\begin{pmatrix} \sqrt{3} & 0 & 0 & 0 & 0 & -\sqrt{6} \\ 0 & \sqrt{3} & 0 & -\sqrt{6} & 0 & 0 \\ 0 & 0 & \sqrt{3} & 0 & \sqrt{6} & 0 \\ 0 & \sqrt{6} & 0 & \sqrt{3} & 0 & 0 \\ 0 & 0 & -\sqrt{6} & 0 & \sqrt{3} & 0 \\ -\sqrt{6} & 0 & 0 & 0 & 0 & -\sqrt{3} \end{pmatrix}$$



Post-select ($\sin^2\theta = 1/3$); success probability = $1/9$

CZ Example - Heralded



Heralded ($\psi = \pi$, $\theta_1 = -\theta_2 = \frac{54.74}{180}\pi$, $\theta_3 = \frac{17.63}{180}\pi$)
Success probability = $2/27$

Problem (General)

Given a quantum computing unitary operation, U, that acts on q qubits, find a photonics circuit, $\hat{U}$, that implements U on its Fock space $U_{\mathcal{F}}$.

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Problem (Optimisation)

Given a quantum computing unitary operation, U, that acts on q qubits, $2q$ (dual-rail) $+\ m_a$ (auxiliary) optical wires and a coincidence basis, $\mathcal{C}$; find a dual-rail linear optics operation $\hat{U}$ that maximises the likelihood of U occurring.

SMT Solvers and dReal

Satisfiability Modulo Theory (SMT)

$$(a + 2b < 5 \vee 3a - b < -1) \wedge (3a - b \geq -1 \vee a + 6 > 10) \wedge \ldots,$$

- Have a dedicated theory solver that can find appropriate values for variables

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- Many software tools available: Z3, Yices, cvc5, ...

δ-Weakening

Non-linear Expressions

$$ab, a^2, a^3b^2, \ldots$$

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dReal - δ-weakening SMT solver

Search Algorithm for Photonics Problem

Finding a δ-sat

$$\texttt{core} = \texttt{bound} \wedge \texttt{unitary} \wedge \texttt{fockequal}$$

$$\texttt{extra constraints}$$

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$\longrightarrow$

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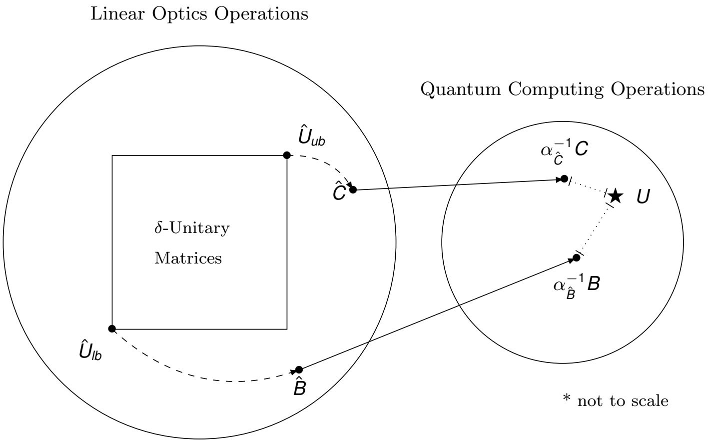
δ-unitary matrices

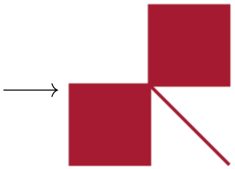
unkown/unsat

Unitary Extraction



* not to scale

Search Algorithm



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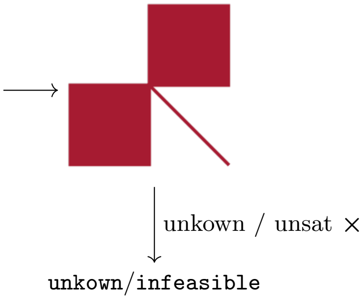

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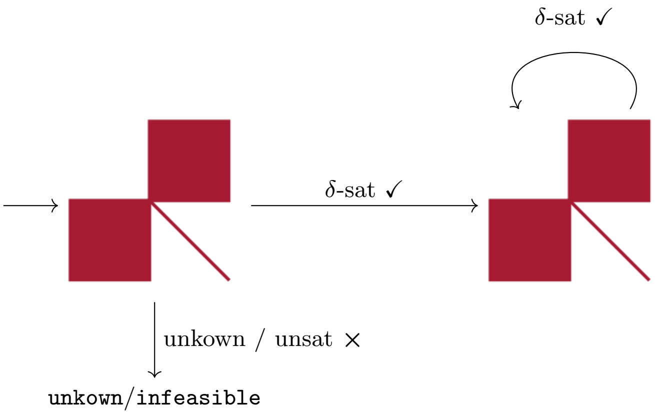

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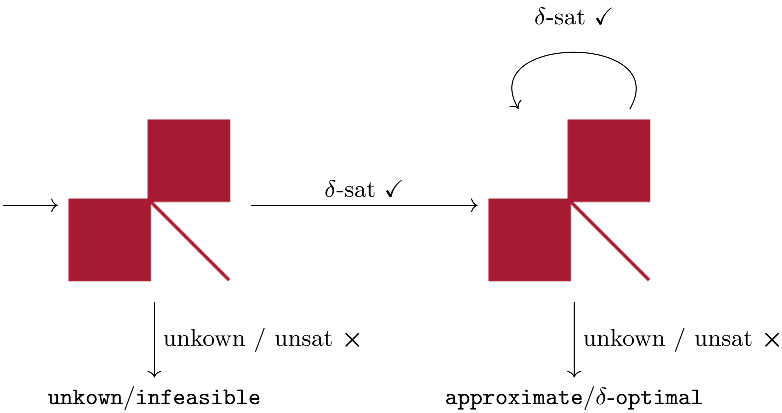

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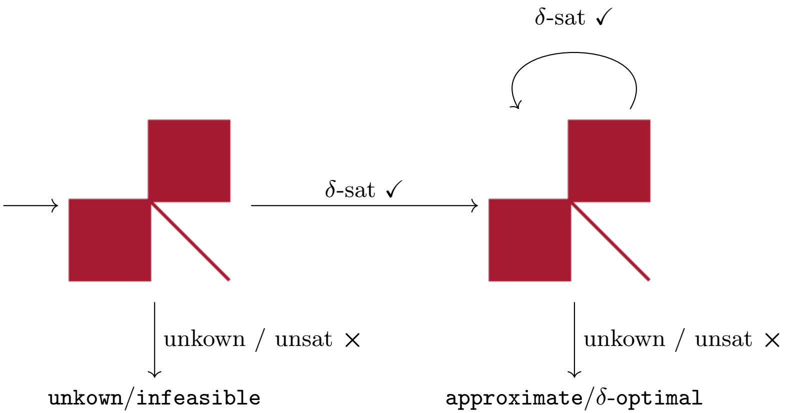

Post-selection: Known Results

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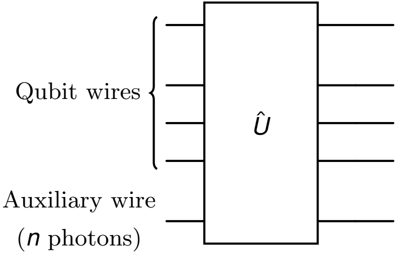
- Similar results for *CNOT*

Post-selection: CZ with one auxiliary wire



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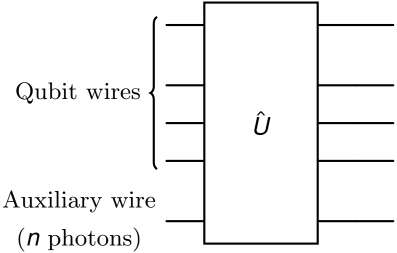

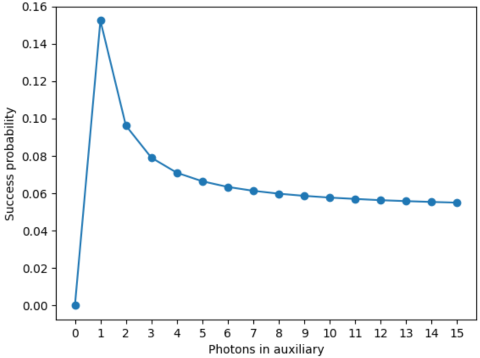

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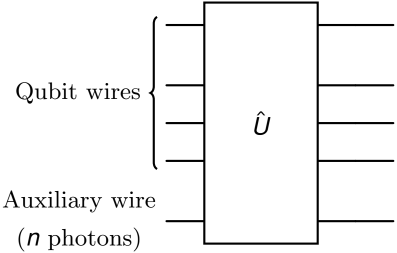

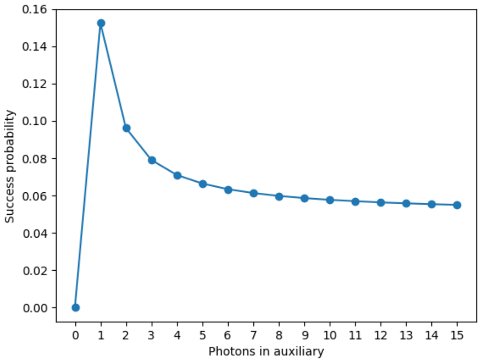

Matches results in Alessio Baldazzi and Lorenzo Pavesi. "Universal multiport interferometers for post-selected multi-photon gates." 2024

Post-selection: Givens Rotation Gates

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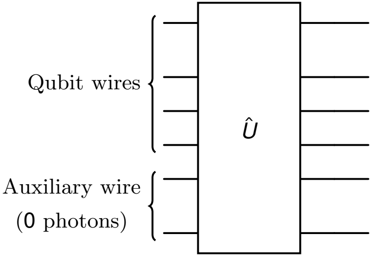

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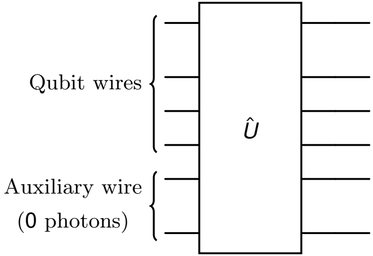

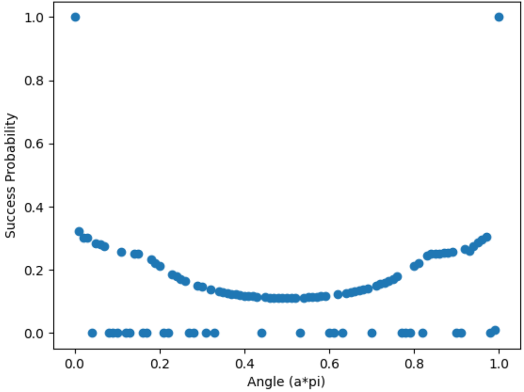

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- *CZ* with two wires and two photons - `unkown` (but known in literature)

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- Finding bounds to optimal result (based on δ)

Thanks for Listening

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